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*FEEDING EXPERIMENTS WITH MIXTURES OF FOODSTUFFS IN UNUSUAL PROPORTIONS**

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Ever since Magendie[1] announced, in 1816, that the protein group of foodstuffs represents an indispensable component of the dietary of the higher animals, students of nutrition have been interested in the part played by the other nutrients that commonly enter into our food intake. Are fats or carbohydrates or both equally essential for successful nutrition? We pointed out some time ago that the reason why there is no available information respecting the actual requirement of the healthy mammal for fat is attributable to the experimental difficulties heretofore inherent in its solution.[2] Until the significance of the accessory food factors now known as vitamines was appreciated studies of the physiological value of mixtures of different foodstuffs, etc., were liable to lead to failure and erroneous conclusions, not because the supply of energy or protein or salts was inadequate but because other unrecognized and unidentified essentials were lacking.

Since it has become possible, by taking account of these various newly appreciated properties of foods, to devise rations in which essentially one factor at a time may be altered, the investigation of the rôle of the individual nutrients has become more promising. Accordingly we have already succeeded in raising young animals (albino rats) from an early age to adult size on diets which were exceptionally poor in fats.

The food mixtures consisted of the residues from extracted lean meat as a source of protein, starch, inorganic salts, together with small quantities of alfalfa and dried brewery yeast furnishing the vitamins A and B. Analyses of the rations consumed indicated that the maximum intake of fat at any time did not exceed 0.3 per cent of the food eaten. Inasmuch as all the animals starting on the diet with a body weight of approxi-

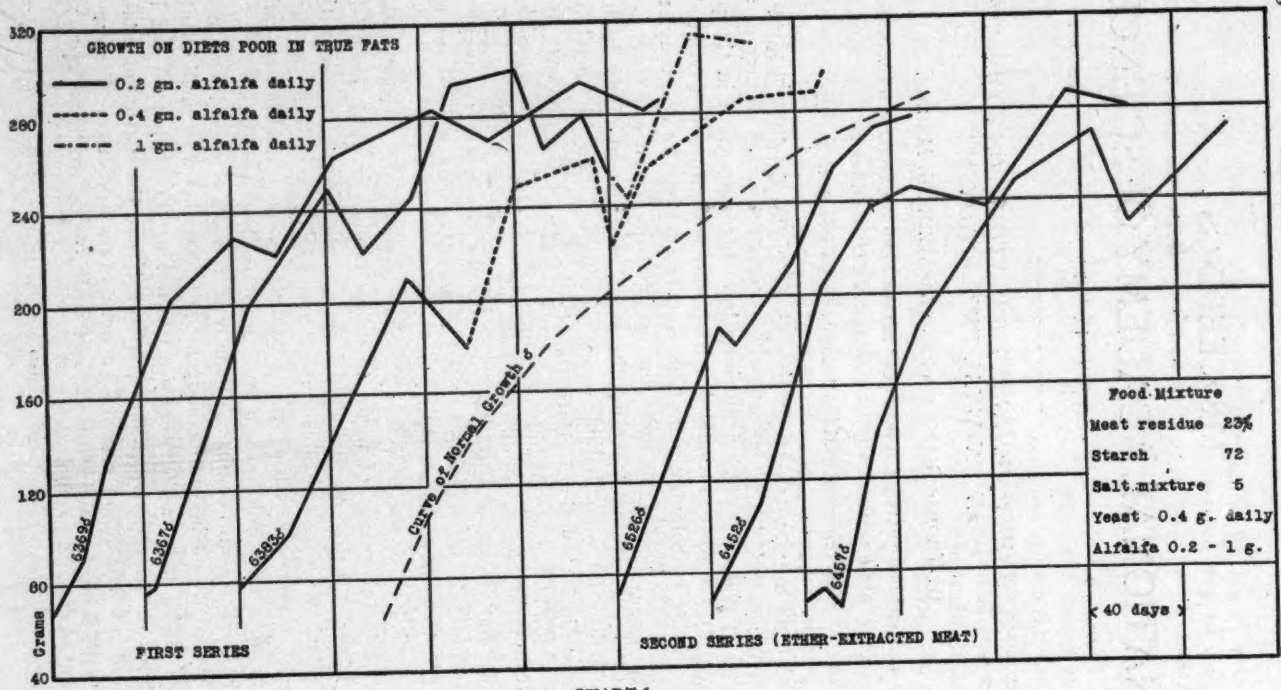
GROWTH ON DIETS POOR IN TRUE FATS
0.2 gm. alfalfa daily
0.4 gm. alfalfa daily
1 gm. alfalfa daily
320
280
240
200
160
120
80
40
Grams
6359♂
6357♂
6383♂
Curve of Normal Growth ♂
6552♂
6452♂
6461♂
FIRST SERIES
SECOND SERIES (ETHER-EXTRACTED MEAT)
Food Mixture
Meat residue 23%
Starch 72
Salt mixture 5
Yeast 0.4 g. daily
Alfalfa 0.2 - 1 g.
< 40 days >

CHART 1

mately 70 gm. have quadrupled their weight within the usual time, and appear as well nourished as companion rats on diets containing liberal portions of butter fat or lard, we cannot avoid the conclusion that if true fats are essential for nutrition during growth the minimum necessary must be exceedingly small. Drummond[3] has also secured some success on diets practically devoid of fat with rats receiving an approximate daily intake of neutral fat amounting to 14 mg. He too concludes that unless this minute amount of fat plays as important a rôle in the metabolism of the organism as do the minute quantities of such substances as the accessory factors, it is reasonable to suggest that pure fats are dispensable constituents of the mammalian diet.

The outcome of all these investigations leads one to question seriously the contentions made, particularly during the recent war, that fats as such play some unique rôle in maintaining well-being; and further, as Maignon[4] supposes, that they play an important rôle in the utilization of protein,—a rôle which carbohydrates are powerless to fill. On the other hand the demonstrations afforded by our experiments should not be construed to minimize the great value of fats as a source of energy in the usual dietary, as well as their peculiar advantages in culinary procedures.

As we have recently pointed out[5] carbohydrates are ordinarily regarded as indispensable components of the food intake. This belief is based on the presence of more or less carbohydrate in the food mixtures consumed by man and the higher animals, and the fact that sugar is a constant constituent of the blood. It is almost universally taught that carbohydrate is essential for the proper metabolism of fats in addition to any other functions that it may perform in the body; for ketone substances may be excreted in diabetes when sugar fails to be burned up in the normal manner in the organism. On the other hand it is assumed that glucose can be formed from the protein molecule or its amino-acids under certain conditions in the metabolism so that one could conceive carbohydrate to become available for the special needs of fat metabolism and other purposes without being specifically furnished as *preformed* carbohydrate in the diet. The current opinion, summarized by one recent writer,[6] maintains that "carbohydrates are the most economical of the foodstuffs, both physiologically and financially. They are the greatest sparers of protein. Ingestion of fat has for its object the relieving of the intestine from excessive carbohydrate digestion and absorption. Ingestion of fat in too large quantities leads to digestive disturbances, and if carbohydrates are entirely abandoned, to acetonuria."

Not long ago we announced that rats receiving a diet in which the amount of digestible carbohydrate was at most exceedingly small can grow from an early age to adult size. The rations which we fed included pro-

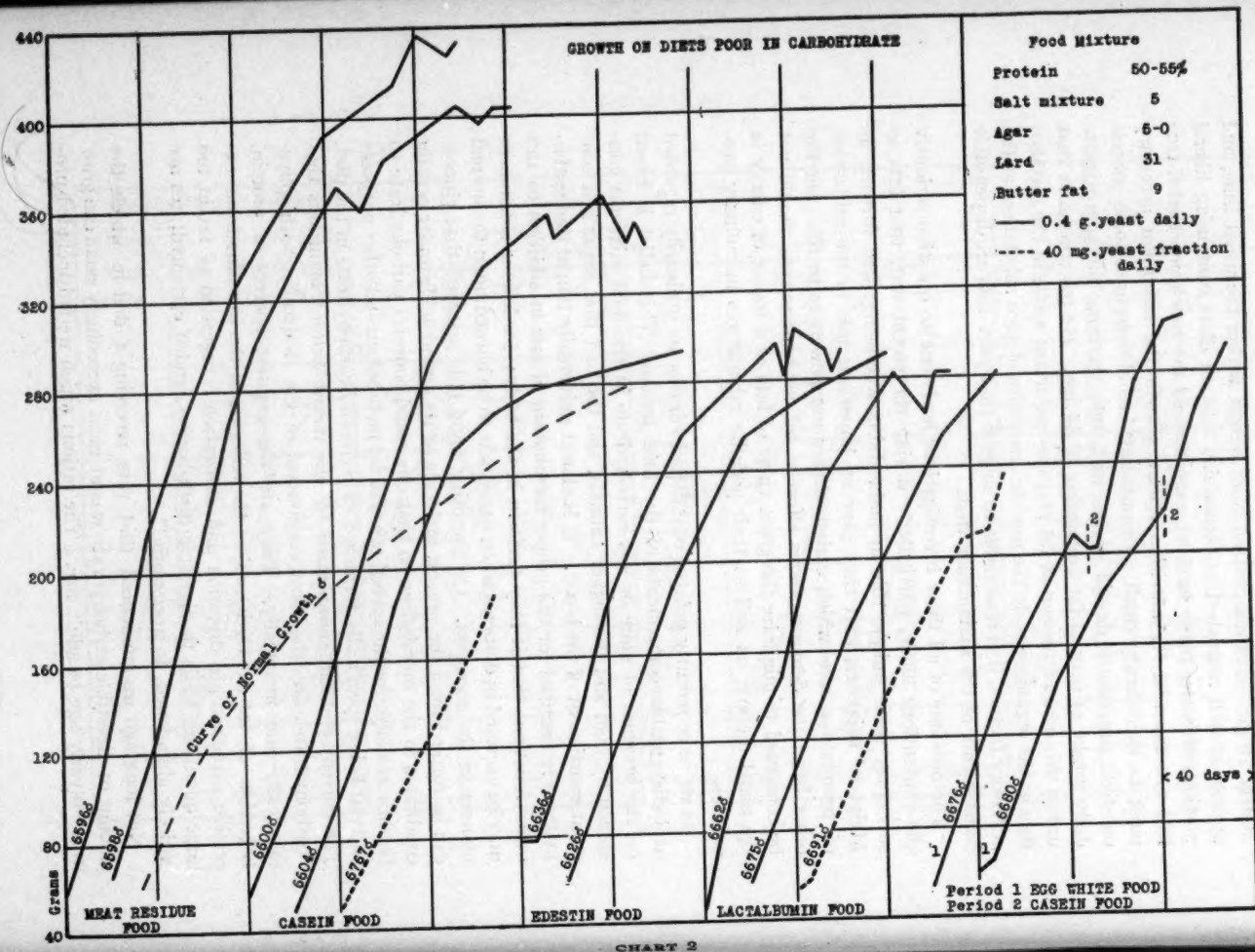
GROWTH ON DIETS POOR IN CARBOHYDRATE
Food Mixture
Protein 50-55%
Salt mixture 5
Agar 5-0
Lard 31
Butter fat 9
—— 0.4 g. yeast daily
---- 40 mg. yeast fraction daily
Grams
440
400
360
320
280
240
200
160
120
80
40
Curve of Normal Growth
6596♂
6598♀
6600♂
6604♂
6718♀
6636♀
6638♂
6682♀
6678♂
6691♂
6676♂
6680♂
1
2
< 40 days >
MEAT RESIDUE FOOD
CASEIN FOOD
EDESTIN FOOD
LACTALBUMIN FOOD
Period 1 EGG WHITE FOOD
Period 2 CASEIN FOOD

CHART 2

tein—casein, edestin, or lean beef which had been thoroughly extracted with boiling water—inorganic salts, agar-agar, lard, butter fat and 0.4 gm. daily of dried brewery yeast furnishing vitamin B. This amount of yeast can scarcely be regarded as a significant source of available carbohydrate. Success was likewise attained in experiments in which no agar-agar was introduced. In the latter case the only obvious sources of preformed carbohydrate aside from the yeast employed were such carbohydrate impurities as might still adhere to the protein preparation fed.

The newest experiments conducted with materials from which the carbohydrate was even more rigorously excluded have not yet been brought to a conclusion; but in these also considerable growth at a normal rate has already been attained. It will be observed that the dietary conditions under which this series of nutrition experiments has been conducted are such as are reputed to lead to ketosis and ketonuria in human subjects.

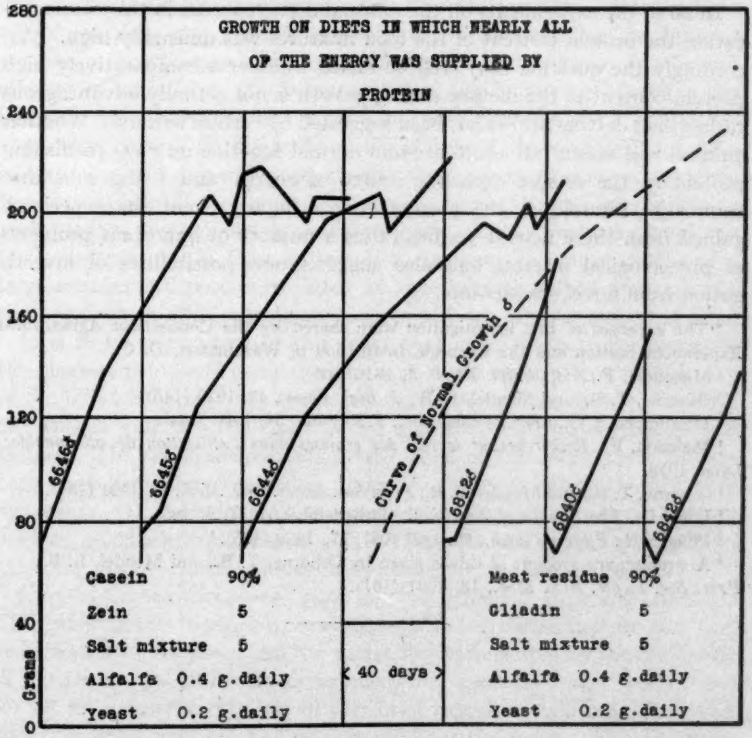

CHART 3

In view of the success of the foregoing experiments we have investigated the possibility of nourishing animals on food mixtures from which both carbohydrates and fats have been eliminated as far as possible. The attempts have thus far resolved themselves essentially into feeding a diet of which more than 90 per cent consists of protein, along with 5 per cent of inorganic salts and vitamins in the form of small daily doses of dried alfalfa and brewery yeast.

Although it has been demonstrated by experiments of Pflüger[7] that a carnivorous animal can be kept alive and maintained in activity of considerable periods on an exclusive diet of meat, it has been stated that omnivora and herbivora cannot survive on such a ration. In our own trials[8] in which the vitamin-bearing substances, representing 4–8 per cent of the food eaten, were the only noteworthy sources of either fat or carbohydrate, a number of animals have already approximately tripled their size, growing at essentially a normal rate for their species.

In all of the experiments on the unusual diets reported in this communication the protein content of the food mixtures was unusually high. Accordingly the question may well be raised whether a comparatively high protein content in the dietary during growth is not actually advantageous rather than detrimental as has been suggested by certain writers. Whether animals will attain full adult size and normal function on diets furnishing protein as the almost exclusive source of energy and tissue substance cannot be foretold at the present time. In any event the experience gained from these newest studies raises a number of important problems of physiological interest and also suggests new possibilities of investigation from novel standpoints.

* The expenses of this investigation were shared by the Connecticut Agricultural Experiment Station and the Carnegie Institution of Washington, D. C.

[1] Magendie, F., *Ann. Chim. Phys.*, **3**, 1816 (66).

[2] Osborne, T. B., and Mendel, L. B., *J. Biol. Chem.*, **45**, 1920 (145).

[3] Drummond, J. C., *Proc. Physiol. Soc., J. Physiol.*, **54**, 1920 (xxx).

[4] Maignon, F., *Récherches sur le rôle des graisses dans l'utilisation des albumnoides.* Lyon, 1919.

[5] Osborne, T. B., and Mendel, L. B., *Proc. Soc. Exper. Biol. Med.*, **18**, 1921 (136).

[6] Lusk, G., *The Science of Nutrition.* Philadelphia, 1917, p. 288.

[7] Pflüger, E., *Pflüger's Arch.*, **50**, 1891 (98); **77**, 1899 (425).

[8] A preliminary account of this is given by Osborne, T. B., and Mendel, L. B., *Proc. Soc. Exper. Biol. Med.*, **18**, 1921 (167).

VARIATION AND INHERITANCE IN SIZE IN TRYPANOSOMA LEWISI

II. THE EFFECTS OF GROWING "PURE LINES"[1] IN DIFFERENT VERTEBRATE AND INVERTEBRATE HOSTS AND A STUDY OF SIZE AND VARIATION IN INFECTIONS OCCURRING IN NATURE

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In our first report[2] we demonstrated that Trypanosoma lewisi reaches an adult stage in the course of the infection in the rat in about 25 days, and once this stage is reached there is practically no division or growth. In consequence of the elimination of growth factors from the computations it was found that the coefficient of variation in the "pure line" was very low if it was computed during this adult period. The questions which now arise are: (1) Does growing the same "pure line" in different vertebrate hosts cause significant differences in the mean or the coefficient of variation? (2) Does passage of the "pure line" through the invertebrate host cause any significant differences in these constants? Our attention will be directed especially toward the coefficient of variation as this will give us an opportunity to ascertain whether or not there is a splitting up of the "pure line" into heritably diverse lines following such passage through the invertebrate host. With this background our final question is: (3) Does an infection occurring in nature consist of a large number of "pure lines" such as have been described in free-living protozoa, which differ among themselves but are *per se* constant in size?

It is to be emphasized at this point that in all of the following work the measurements and computations were made on or after the 30th day of the blood infection. As the adult stage is always reached by the 25th day this is well into the adult period and effectually eliminates growth factors. The six measurements of size used in this work have already been given in our first report (see figure 1). They consist of the following distances: (1) Posterior end *to* parabasal body, (2) parabasal body *to* nucleus, (3) nucleus *to* anterior end, (4) anterior end *to* end of flagellum, (5) total length which includes measurements 1–4, and (6) width.

Effect of Growing the Same "pure line" in Different Vertebrate Hosts.—The twoconstants in which we are interested in this connection are the size as indicated by the mean and the variability as indicated by the coefficient of variation. In all of the experiments the constants were worked out for all six measurements but in this brief report we will only deal with those for total length. Let us first consider the mean. The questions are: Does growing the same pure line in different *individuals* of the same

species of rat produce significant differences in size? Does growing the same "pure line" in different *species* of rats produce any greater differences in size than in the first case of individuals belonging to the same species? It would have been interesting to grow the "pure line" in other vertebrates as well as in rats but T. lewisi is not normally infective to any other vertebrate. To test the first of these questions we used individuals of the albino rat and for the second question we used individuals of the black and the Norway rat. At the outset it may be definitely said that the differences which were obtained on growing the "pure line" in different species of rats were in no way more significant than the differences obtained by growing the "pure line" in different individuals of the same species. The greatest difference in mean size[3] was obtained when the same "pure line" was grown in the albino rats 46 and 90. In the former the mean length was $32.080 \pm .052$ and in the latter $30.769 \pm .057$, which gives us a difference of $1.311 \pm .077$. The mean lengths obtained in twelve other experiments lie between these limits. While a difference of this character, which is 16.8 times its probable error, is undoubtedly significant the question immediately arises as to whether such a difference is due to differences in the blood of different rats or to a personal variation in making the measurements. A definite answer cannot be given to this question at the present time although experiments are being carried out to ascertain, if possible, which is the determining factor. As, however, this is the greatest difference obtained in twelve experiments, we may say that growing the same "pure line" in different rats probably causes significant differences in mean size, but in any case these differences are small. Even if these differences are significant there is no evidence, at present, that they are due to inherited diversities, but are simply due to differences in environment.

In regard to variability, the chief thing of interest is the fact that passage of the "pure line" from rat to rat by blood inoculation has no effect on the coefficient of variation. Take, for example, the following experiment. The coefficient of variation for total length in the "pure line" as grown in rat 116 was $2.77 \pm .13\%$. After four passages through white rats this line was grown in rat 204 where the coefficient of variation was found to be $2.60 \pm .12\%$. The difference between these two coefficients is $.17 \pm .18\%$ which certainly is not significant. Several other experiments gave similar results and an experiment is being carried out now in which the "pure line" is undergoing numerous transfers from one rat to another. The fact that passage of the pure line through rats has no effect on the coefficient of variation is in marked contrast to conditions found after passage through the invertebrate host.

Effect of Passing the "pure line" through the Invertebrate Host.—While T. lewisi can be experimentally transmitted by several species of ecto-

parasites the natural insect vectors are the rat fleas, Xenopsylla cheopis and Ceratophyllus fasciatus. In working out the life history of certain species of trypanosomes some of the earlier investigators described what they considered to be sexual phases in the life history of the parasite. Practically all of these observations are open to entirely different interpretations, so that at the present time, although a number of protozoölogists feel that there is a sexual phase in the life cycle, no such phase has been demonstrated. This is notably true in the classic researches of Minchin and Thomson[4] on the life history of T. lewisi in the rat flea. Although they looked carefully for conjugation they found no evidence of it.

While there is no evidence of sexual phenomena in the trypanosomes during passage through the invertebrate host, there is evidence that such passage exerts a profound effect on such things as acquired physiological characteristics. Gonder[5] for example found that arsenic fastness was transmitted from rat to rat but was lost by passage through the louse. Miss Robertson[6] found that strains of T. gambiense showed marked changes in their characteristics after passage through the tsetse-fly. This led this author to say, "It seems clear that the cycle in the fly as a whole, whether conjugation occurs or not, has much of the biological significance of the process."

As we have seen passage of the same "pure line" through a number of rats does not increase the variability of the "pure line." Passage through the flea, on the other hand, *invariably* increases the variability. These experiments were carried out in the following manner. Fleas which were carefully raised in the laboratory and which were known to be free from infection were allowed to bite animals infected with a given "pure line." After about a week (during which time the trypanosomes were undergoing their development in the flea) other rats were infected either by teasing up a single flea and injecting it intraperitoneally or by allowing the rat to lick up the moist feces, which is the natural mode of infection. The trypanosomes in the rat which was infected from the flea were measured on the 30th day of the blood infection. Six such experiments have been successfully carried out. The following is a fair example of all of them. The pure line in rat 105 showed a coefficient of variation of $2.80 \pm .13\%$ for total length. After passage through a single specimen of X. cheopis it was measured in rat 163. Here the coefficient of variation increased to $5.24 \pm .25\%$, a difference of $2.44 \pm .28\%$. As we have seen above, no such increase in variability has been observed during passage of the "pure line" from rat to rat. From these experiments we must conclude that the "pure line" breaks up during passage through the invertebrate host. This is analogous to the results of Jennings[7] in Paramecium after conjugation and in a recent publication Miss

Erdmann[3] believes that she has demonstrated the same type of phenomena in Paramecium after endomixis. While the increase in variability is probably due to some nuclear phenomena during the life cycle of the trypanosome in the flea, we have no method of judging, from the present data on the subject, whether it is a sexual or a reorganization process. Minchin and Thomson[4] have suggested that the peculiar effects produced by passage through the invertebrate host are connected with the transformation of the trypanosome into a crithidial stage and the reverse process, both of which always take place in the life cycle in the invertebrate host.

It is conceivable that passage of a "pure line" through the invertebrate host might cause significant changes in the mean size as well as in the variability. All of our experiments have shown this not to be true. It is true that after the "pure line" is passed through a flea there is often a difference in the mean which is significant from a statistical standpoint, but as these differences are never greater than the maximum difference given above for growing the "pure line" in different rats, we cannot ascribe the difference to any peculiar effect of the flea. It is probably due simply to the fact that the trypanosomes are measured in different rats.

Size and Variability in "wild"[1] Infections Occurring in Nature.—A number of naturally infected rats were collected from around Baltimore and Washington. Up to the present time measurements have been made of specimens from ten of these infections. While these "wild" infections do show differences in their means and a higher coefficient of variation on the average than the pure lines, one is struck with the constancy of the means and the low variability. The longest mean length (rat 221) obtained is $32.503 \pm .060$ and the shortest (rat 411) $29.093 \pm .062$ with a difference of $3.410 \pm .087$. It can be seen that the difference between the longest "wild" infection and the shortest is comparatively great and that the difference is much greater than is obtained by growing a "pure line" in different rats. The least variable "wild" infection (rat 413) showed a coefficient of variation of only $2.125 \pm .101$. This is slightly lower than any of the coefficients of the "pure lines" in the laboratory. The most variable "wild" infection (rat 65) showed a coefficient variation of $4.583 \pm .223$ The difference between the least and the most variable "wild" infection is 2.45 ± 24. The remainder of the "wild" infection measure fall between the figures given here, both as regards their means and coefficients of variation.

From these results we must conclude that while the "wild" infections occurring in nature show comparatively small differences in size, these differences are greater than can be explained by the fact that the infections are occurring in different rats. We must also conclude that most "wild" infections consist either of very few "pure lines" or a larger number

which differ *inter se* by very minute differences. In some cases, as in that of rat 413, we are almost forced to believe that a given "wild" infection is actually a "pure line." A possible explanation of these facts suggests itself when we consider the mechanism of infection with T. lewisi. As stated above the rat receives the infection from the flea. As the infected flea is feeding it deposits its feces, which contain the infective forms of the trypanosomes, on the fur of the rat. The rat then becomes infected by licking up the *moist* feces containing the infective forms. In nature it is probable that a given rat receives its infection from a single flea. This flea deposits a number of infective forms. The chances are that most of these will be killed by drying and that only a few will actually start the infection in the rat. This being so, the variability of the infections simply reflect the number of infective forms of the trypanosomes that actually start growth in the rat. In fact it is conceivable that a number of the "wild" infections are actually "pure lines."

At the present time the author is of the opinion that if we consider all the infections in nature collectively we can say that there are a large number of "pure lines" which differ among themselves but which (as is shown by work in the laboratory) are *per se* constant in size. Due to the peculiar way in which rats receive their infections, however, we find that any given infection consists of only a very few of these "pure lines."

Conclusions.—At the present phase of the work the following conclusions may be drawn: While growing the same "pure line" in different rats may cause significant differences in the mean size, these differences are small. The differences in the mean are never greater when the "pure line" is grown in different species of rats than when it is grown in different individuals of the same species. Passage of the "pure line" from rat to rat is not followed by any significant changes in the coefficient of variation. In marked contrast to this it is found that passage of the "pure line" through the flea is invariably followed by a significant increase in the coefficient of variation which is interpreted as showing that the "pure line" breaks up into heritably diverse lines following such passage. Although passage through the flea has such a marked effect on the variability it has no significant effect on the mean size. "Wild" infections occurring in nature exhibit differences in their mean sizes which are greater than can be explained by the fact that they occur in different rats. The coefficients of variation of "wild" infections are, on the average, higher than for the "pure line." The lowest of these coefficients, however, is no greater than the coefficient of variation for the "pure line" and even the highest can be explained on the assumption that the "wild" infection consists of only a very few "pure lines." It seems probable that there occur, in nature as a whole, a large number of

"pure lines" which differ among themselves but are *per se* constant in size, but, due to the peculiar way in which rats receive their infections, a given infection consists of only a very few of these lines. In fact, a large number of "wild" infections are probably actual "pure lines."

[1] Throughout this work the term "pure line infection" is used to designate an infection, the trypanosomes of which have all arisen from a single organism. A given "pure line" may either have been started from a single trypanosome, or it may have been subinoculated from such an infection. The term "wild infection" designates an infection as found in nature.

[2] These PROCEEDINGS, 7, 1921 (138–143).

[3] The mean sizes in this report are all in microns.

[4] Minchin, E. A., and Thomson, J. D., *Quart. J. Microsc. Sci.*, 60, *N. S.*, 1915 (463–692).

[5] Gondor, R., *Centralbl. Bakt., etc.*, I abt., Originale, 612, 1911 (102–113).

[6] Robertson, M., *Proc. Roy. Soc.*, (B) 85, 1912 (241–248).

[7] Jennings, H. S., *J. Exper. Zool.*, 11, 1911 (1–134); *Ibid.*, 14, 1913 (270–391).

[8] Erdmann, R., *Archiv. Entwicklungsmech. Organ.*, 46, 1920 (85–148).

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*NOTE ON MOVING EQUILIBRA**

BY ALFRED J. LOTKA
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A number of previous publications[1] have been devoted to the study, from various angles, of a material system evolving in accordance with a system of differential equations

$$dX_i/dt = F_i(X_1, X_2, \ldots; A_1, A_2, \ldots; P; Q) \qquad (1)$$

where the X's denote the masses of the several components of which the system is built up; the A's are parameters introduced by any equations of constraint to which the X's may be subject;[2] the parameters P include geometrical constraints (volume, area, topography) and also other quantities serving to define the state of the system (temperature, etc.). The Q's define the character of the several components or species.

The discussion has hitherto been restricted to the case that the parameters A, P, Q remained constant during the transformations taken in view. A complete discussion of the evolution of systems of the kind referred to must include also the consideration of changes in these parameters.

Such changes may be grouped under three heads:

1. Changes of a perfectly general character. A study of these would resolve itself into a discussion, on a general basis, of a system of differential equations of the form

$$dX_i/dt = F_i(X_1, X_2, \ldots, t) \qquad (2)$$

It is not intended to enter here into this perfectly general case. It will suffice to point to the purely mathematical literature on the subject.[3]

2. Changes in the parameters A, P, Q proceeding very slowly as compared with the speed of readjustment of the X's. In this case the system, after passing through a "transient" state, ultimately settles down to a "moving equilibrium." The study of this case therefore comprises two phases, which may with advantage be taken up separately. The phase of the moving equilibrium is of particular interest, since such moving equilibria play an important rôle in the evolution of physical systems, as pointed out years ago by Herbert Spencer.[4]

3. We may study the effect of a change in a parameter A, P or Q upon the equilibrium of the system alone, irrespective of the process by which that equilibrium is reached. It is in this case immaterial whether the change is slow or rapid. Into this division of the subject fall such relation as the principle of Le Chatelier, the thermodynamic laws of equilibrium and the "reciprocal relations" of statistical mechanics.

The application of some of these principles to biological and social systems has been essayed, but it cannot be said that the rigor of the attempts thus made is satisfactory. It would therefore be desirable to go over the ground and consolidate it. An effort in this direction is taken in view as part of the plan into which the present contribution is fitted.[5]

Of the general field outlined above, the portion to which we shall now give our attention is that of moving equilibria.

Our fundamental system of equations we shall, for our present purpose, write in the form (2). Furthermore, merely in order to simplify our notation, we will restrict the number of dependent variables to two, which, to avoid subscripts, we will denote by X, Y. We have, then

$$dX/dt = F_1(X, Y, t), \qquad dY/dt = F_2(X, Y, t). \tag{3}$$

We adopt a method of successive approximations. Since the system is near equilibrium, we write for our first approximation

$$0 = F_1(X, Y, t), \qquad 0 = F_2(X, Y, t). \tag{4}$$

Solving for X and Y we then have

$$X_1 = \varphi_1(t), \qquad X_1 = \psi_1(t). \tag{5}$$

whence by differentiation

$$dX_1/dt = X_1' = \varphi'_1(t), \qquad dY_1/dt = Y'_1 = \psi'_1(t). \tag{6}$$

Proceeding to a second approximation, we substitute (6) in (3).

$$\varphi'_1(t) = F_1, \qquad \psi'_1(t) = F_2. \tag{7}$$

Solving again for X, Y,

$$X_2 = \varphi_2(t), \qquad Y_2 = \psi_2(t.) \tag{8}$$

We may again differentiate,

$$X'_2 = \varphi'_2(t), \qquad Y_2 = \psi'_2(t). \tag{9}$$

and, substituting as before, we obtain a third approximation. We continue this process as far as may be desired, and finally obtain for the *n th*

approximation

$$X_n=\varphi_n(t), \qquad Y_n=\psi_n(t). \tag{10}$$

If the functions φ_n, ψ_n, and φ'_n, ψ'_n, obtained by these successive approximations tend toward a limit as n is increased indefinitely, then it can easily be shown that (10) is a solution of (3).

It seems to be somewhat difficult to give in general terms the conditions for this convergence of successive approximations toward a limit. However, the following example will show how in individual cases the question regarding this convergence can sometimes be readily answered.

For our example we take the case of radioactive equilibrium. We have a chain of transformations

$$A \longrightarrow B \longrightarrow C \longrightarrow D \tag{11}$$

We will denote the masses of A, B, C, D at time t, respectively, by U, X, Y, Z, and their values at time $t=0$ by the same letters with the subscript zero.

We may if we choose (this is a purely arbitrary matter) pick out the substances B and C for our evolving system, and look upon A and D as external factors influencing the system. The system composed of B and C is then subject to an equation of constraint

$$X+Y=X_0+Y_0+U_0-U+Z_0-Z \tag{12}$$

in which the quantities appearing in the right hand member are parameters of the class denoted above in equation (1) by A, that is to say, parameters introduced by the equation of constraint. Four of them are constants, the other two are functions of t. Of these last two one, namely U, will appear in the equations representing the course of evolution of the system composed of B and C. These equations, according to the well-known laws of radioactive transformations are

$$dX/dt=aU-bX, \qquad dY/dt=bX-cY. \tag{13}$$

where a, b, c are constants. U, on the other hand, is a function of the time, namely

$$U=U_0e^{-at} \tag{14}$$

For our first approximation we put, then,

$$dX/dt=0=aU_0e^{-at}-bX_1 \qquad dY/dt=0=bX_1-cY_1 \tag{15}$$

whence

$$X_1=a/b.\ U_0e^{-at}, \qquad Y_1=a/c.\ U_0e^{-at}, \tag{16}$$

and therefore

$$X'_1=-a^2/b.\ U_0e^{-at}, \qquad Y'_1=-a^2/c.\ U_0e^{-at}. \tag{17}$$

The second approximation now gives

$$dX/dt=X'_1=-a^2/b.\ U_0e^{-at}=aU_0e^{-at}-bX_2 \tag{18}$$

$$dY/dt-Y'_1=-a^2/c.\ U_0e^{-at}=bX_2-cY_2 \tag{19}$$

Whence

$$X_2=a/b.\ U_0e^{-at}\ (1+a/b) \tag{20}$$

$$Y_2=a/c.\ U_0e^{-at}\ (1+a/b+a/c) \tag{21}$$

Continuing this process we find, ultimately,

$$X_n = a/b\ U_0 e^{-at}\ (1 + a/b + a^2/b^2 + \dots) \quad (22)$$

$$Y_n = a/c\ U_0 e^{-at}\ (1 + a/b + a/c + a^2/b^2 + a^2/bc + a^2/c^2 + \dots) \quad (23)$$

In this example the condition for the convergence of the successive approximations is immediately apparent. We must have

$$a/b < 1, \qquad a/c < 1; \quad (24)$$

that is to say, the parent substance A must have a smaller decay constant than any of the succeeding members of the series. For obvious reasons this condition is always satisfied in natural radioactive mixtures.[6]

The series (22), (23) bring out the relation between the uncorrected equilibrium, as commonly computed on the assumption of constancy of mass of the parent substance, and the true equilibrium. The first-mentioned (for which Rutherford has suggested the term "secular equilibrium") is represented by the first term of the series. As Rutherford points out, the error of the first approximation, i.e. the difference between the secular and the true equilibrium, amounts, in some cases, to nearly 1% though in others the error is quite negligible.

The series are easily summed, and then lead to the well-known expressions obtained by other methods (for the equations of radioactive change are readily integrable in finite terms, while the method here developed is applicable also in more refractory cases).

The case of radioactive equilibrium was here selected as an illustration, primarily because the functions involved are known and of simple form. But the same example will serve very aptly to illustrate also some other points.

In the first place we observe that moving equilibria might be divided into three classes, according as their progress is determined by a change in the P's, the Q's or the A's. As has been shown, the radioactive equilibrium is of the type in which the pace is set by a parameter of the class A, namely *the mass of one of the links in the chain*, which thus acts as a brake, or a *limiting factor* checking the series of transformations. Such limiting factors play an important rôle also in the highly complex network of interlocking cycles upon which the continuance of abundant life upon the earth depends. For life processes are energy transformation processes carried out by the agency of material energy transformers. Such transformers, if they are to work continuously and indefinitely must perforce work in closed transformation chains or cycles (such as the cycle $CO_2 \rightarrow$ Plant $\rightarrow$ Animal $\rightarrow CO_2$). The moving equilibria engendered in such systems of cycles by a slow change in a limiting factor, in a parameter of class A, invite further study. The influence of man upon the world's events seems to have been largely to accelerate the circulation of matter and energy through such cycles, either by "enlarging the wheel", i.e., increasing the mass taking part in certain cycles, or else by causing it to

"spin faster," i.e., increasing the velocity of the circulation, decreasing the time required for a given mass to complete the cycle. In either case he has increased the energy turn-over per unit of time. Whether, in this he has been unconsciously fulfilling one of those laws of nature according to which certain quantities tend toward a maximum, is a question well deserving of our attention.

* Papers from the Department of Biometry and Vital Statistics, School of Hygiene and Public Health, Johns Hopkins University, No. 44.

[1] Lotka, A. J., *Physic. Rev.*, **24**, 1912 (235–238); *J. Washington Acad. Sci.*, **2**, 1912 (2, 49, 66); *Science Progress*, **55**, 1920 (406–417); *Proc. Am. Acad. Arts Sci.*, **55**, 1920 (237–153); these PROCEEDINGS, *Sci.* **6**, 1910 (410–415).

[2] *Proc. Amer. Acad.*, loc cit., p. 142.

[3] See for example Picard, *Traité d'Analyse;* H. Bateman, *Differential Equations*, 1918, p. 245.

[4] Spencer, *First Principles*, Chapter XXII; Winiarskie, "Essai sur la Mécanique Sociale," *Revue Philosophique*, **44**, 1900 (113).

[5] At the time of reading proof this project is partially realized. A discussion of the applicability of the Le Chatelier principle to systems of the general character here considered will appear in a forthcoming issue of the *Proceedings of the American Academy of Arts and Sciences*

[6] Lotka, A. J., *London, Phil. Mag.*, Aug., 1911, p. 353.

A FORMULA FOR THE VISCOSITY OF LIQUIDS[1]

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1. In this paper I obtain for the viscosity of a liquid the formula

$$\eta = \frac{n N h}{2M(v-\delta)}, \qquad (1)$$

where N is the number of molecules in a mol, h is Planck's constant, M is the molecular weight of the liquid in the gas phase, v its volume per gram, and n an integer. The quantity δ is the co-volume as used in the equation of state of Keyes[1]

$$p = \frac{RT}{v-\delta} - \frac{A}{(v-l)^2}, \qquad (2)$$

In all the cases to which I have applied the formula, $n=6$ and so (1) takes the form

$$\eta(v-\delta) = 3Nh/M \qquad (3)$$

It is to be noted that $3N/M$ is the number of translational degrees of freedom of the molecules in the volume v of the liquid.

2. To prove equation (1), let x, y, z be rectangular coördinates and suppose the liquid to flow parallel to the x-axis in such a way that, u_0 being

the velocity at the point (x, y, z),

$$\partial u_0/\partial z = 1. \tag{4}$$

Let the components of velocity of a molecule parallel to the x and z axes be u, w respectively. When a molecule of mass m moves in the positive direction of the z-axis across the xy-plane, the x-component of momentum transferred across that plane is mu. When it crosses in the negative direction the transfer is $-mu$. Under these conditions the viscosity η of the liquid is defined as the total x-component of momentum transferred across unit area of the xy-plane in unit time.[2] To find the viscosity we therefore find the number of molecules of each velocity crossing unit area and multiply by the average momentum transferred by each.

Let $f(w)\,dw$ be the number of molecules per cubic centimeter with components of velocity parallel to the z-axis between w and $w + dw$. If the molecules did not influence each other, the number of these that would cross unit area of the xy-plane in unit time would be

$$w f(w)\,dw. \tag{5}$$

Because of the interference of molecules with each other, this must be replaced by

$$\frac{v}{v-\delta} w f(w)\,dw. \tag{6}$$

To see this we note that if the molecules moved independently, the z-component of momentum transferred across unit area of the xy-plane in both directions in unit time would be the pressure

$$RT/v. \tag{7}$$

From equation (2), in the real liquid this is replaced by

$$RT/(v-\delta) \tag{8}$$

Now the mass of the molecule and the temperature (average kinetic energy) are the same in both cases. Hence the increased momentum (8) compared with (7), can only be due to an increase in the number of molecules crossing in unit time. Since the same law of distribution of velocities (Maxwell's Law) is assumed in both cases, the ratio in which the numbers are increased must be the same $v/(v-\delta)$ for all velocities. Thus the number of molecules with velocity components between w and $w + dw$ crossing unit area of the xy-plane in unit time is given by equation (6).

The velocity u of a given molecule parallel to the x-axis will not always be equal to the average velocity u_0 of the liquid at that point. There will, however, be a series of instants (which we may call collisions) when u will be equal to u_0. I assume that the interval between two such instants will be a half period (interval between libration limits)[3] in the sense of the quantum theory, and that the molecules can be treated as moving with constant velocity between consecutive collisions. Suppose then a molecule reaching the xy-plane has been moving a time, t, since its last collision.

From (4) its momentum parallel to the x-axis due to the motion of the liquid at that point will be $m\ w\ t$.

From (6) the total x-component of momentum transferred in the positive direction across unit area of the xy-plane in unit time will then be

$$\frac{v}{v-\delta}\int_0^\infty m\ w^2 t f(w)\,dw.$$

There will be an equal transfer due to molecules moving in the negative direction. Hence

$$\eta = \frac{v}{v-\delta}\int_0^\infty 2m\,w^2\,t\,f(w)dw. \tag{9}$$

The time t can be considered as the interval between collisions at times[4] t_0, t_1. Since the molecule is assumed to move with constant velocity between collisions,

$$2m\,w^2\,t = 2\int_{t_0}^{t_1} m\,w^2\,dt = nh, \tag{10}$$

where, according to Sommerfeld's theory, n is an integer. Also

$$v\int_0^\infty f(w)\,dw = N/2M, \tag{11}$$

since it is the number of molecules per gram for which w is positive. Combining (9), (10), and (11), we get (1) which was to be proved.

3. Owing to lack of data on the equation of state, the only substances on which the formula can at present be tested are carbon dioxide, ether, and mercury. Using the values of N and h given by Birge,[5]

$$3Nh = 3(6.0594)(6.5543)10^{-4} = .011914,$$

and so equation (3) takes the form

$$\eta(v-\delta) = .011914/M. \tag{12}$$

Values of δ for carbon dioxide and ether,[6] tables I and II, were supplied by Professor Keyes. At low temperatures the measured viscosities and those calculated by equation (12) do not differ by more than the experimental error. In case of carbon dioxide the temperature 30° is too near the critical point ($t = 31°$) for satisfactory use of the equation of state. Above 10° the calculated viscosity of ether is too large, the difference increasing with the temperature. This may be due to the fact that ether is a complex of more than one type of molecule. The equation of state was determined on the assumption that each liquid molecule is formed by the combination of two gas molecules. This may be substantially true at 10° and not at 100°.

To obtain values of $v-\delta$ for mercury, I make use of the fact that monatomic substances seem to have constant co-volumes[7] δ. Since mercury is monatomic in the gas phase, I assume that δ is constant or nearly constant in the liquid phase. Also the term

$$RT/(v-\delta)$$

in case of mercury is very large (more than 15000 atmospheres). Hence at atmospheric pressures we may neglect p and so write (2) in the form

$$\sqrt{\frac{V-\delta}{T}} = c\,(v-1)$$

where c is constant. Differentiating this equation with respect to v and then with respect to T, assuming δ constant, we get

$$(v-\delta)^2\left(3\frac{dv}{dt}+2T\frac{d^2v}{dt^2}\right)-2T(v-\delta)\left(\frac{dv}{dt}\right)^2-T^2\left(\frac{dv}{dt}\right)^3=0,$$

whence

$$V-\delta=T\frac{dv}{dt}\left[\frac{\frac{dv}{dt}+\sqrt{4\left(\frac{dv}{dt}\right)^2+2T\frac{d^2v}{dt^2}}}{3\frac{dv}{dt}+2T\frac{d^2v}{dt^2}}\right]. \qquad (13)$$

According to Callendar and Moss[8]

$$\frac{V}{V_0}=1+1805553\left(\frac{t}{100}\right)10^{-8}+12444\left(\frac{t}{100}\right)^2 10^{-8}+2539\left(\frac{t}{100}\right)^3 10^{-8} \qquad (14)$$

is the ratio of the volume of mercury at $t°$ to its volume at 0°. The values of $v-\delta$ in table III are obtained by calculating the derivatives of v from (14) and substituting in (13). Below 100° the difference of the calculated viscosities and those measured is not greater than the difference between the values obtained by different observers. At higher temperatures the calculated viscosities are consistently smaller than those measured. The observations, however, differ greatly, owing probably to oxidation of the mercury in contact with the air. The result may be to increase the viscosity by almost any amount. This work should be repeated with mercury of the greatest purity in a vacuum or in contact with some inactive gas.

TABLE I

CARBON DIOXIDE

$$\log\ \delta = .09780 - \frac{0.1978}{v},\quad \eta(v-\delta) = .0002708$$

TEMP.	PRESSURE	v	$v-\delta$	$\eta\times10^6$ CAL.	$\eta\times10^6$ OBS.	OBSERVER
5	Sat.	1.113	.278	974	925	Warburg and Babo[9]
10	"	1.168	.317	854	852	
15	"	1.228	.360	751	784	
20	"	1.306	.419	646	712	
20	59 at.	1.291	.408	664	697	Phillips[10]
20	72	1.224	.358	756	771	
20	83	1.189	.332	816	823	
30	72	1.566	.627	432	458	
30	90	1.339	.445	608	643	
30	104	1.274	.395	686	733	

TABLE II

ETHER

$$\log \delta = .48959 - \frac{.50981}{v}, \quad \eta(v-\delta) = .0001608$$

TEMP.	v	$v-\delta$	$\eta \times 10^6$ CAL.	$\eta \times 10^6$ OBS.	OBSERVER
0°	1.3583	.0573	2806	2860	Thorpe and Roger[11]
10	1.3797	.0612	2626	2585	
20	1.4016	.0663	2425	2345	
30	1.4247	.0703	2287	2120	
30	1.4247	.0703	2287	2134	Pound[13]
30	1.4247	.0703	2287	2150	Heydweiller[13]
40	1.4506	.0761	2113	1970	
50	1.4784	.0828	1942	1810	
60	1.5019	.0888	1811	1666	
70	1.5309	.0961	1673	1528	
80	1.5620	.1058	1519	1400	
90	1.6000	.1177	1366	1284	
100	1.6380	.1302	1235	1177	

TABLE III

MERCURY

$$\eta(v-\delta) = .00005935$$

TEMP.	$v-\delta$	η-CAL.	η-OBS.	$\eta(v-\delta)$	OBSERVER
−18.1	.0033282	.01784	.01836	.00006111	Koch[14]
0	.0035606	.01667	.01661	.00005914	Plüss[15]
0	.0035606	.01667	.01688	.00006010	Koch
10	.0036885	.01609	.01577	.00005817	Umani
10.1	.0036897	.01609	.01620	.00005975	Koch
14.95	.0037538	.01583	.01571	.00005898	Benard[16]
15.1	.0037557	.01581	.01575	.00005914	"
15.7	.0037635	.01578	.01570	.00005909	"
15.9	.0037661	.01577	.01583	.00005961	"
15.95	.0037668	.01577	.01581	.00005954	"
16.1	.0037687	.01576	.01563	.00005889	"
16.2	.0037700	.01575	.01563	.00005892	"
16.5	.0037717	.01574	.01572	.00005929	"
18.76	.0038006	.01562	.01558	.00005921	Plüss
20	.0038165	.01555	.01547	.00005904	"
20	.0038165	.01555	.01579	.00006026	Schweidler
34.05	.0039967	.01485	.01476	.00005899	Plüss
40	.0040732	.01457	.01483	.00006041	Schweidler
62.9	.0043682	.01359	.01360	.00005941	Plüss
79.4	.0045817	.01295	.01299	.00005952	"
97.95	.0048225	.01230	.01263	.00006091	"
99	.0048362	.01227	.01227	.00005934	Koch
124	.0051589	.01151	.01171	.00006041	"
137.5	.0053304	.01114	.01167	.00006221	Plüss
154	.0055456	.01070	.01092	.00006056	Koch
			av.	.00005968	

[1] F. G. Keyes, "A New Equation of Continuity," *Proc. Nat. Acad. Sci.*, 3, 1917 (323–330).

[5] Jeans, *Dynamical Theory of Gases*, p. 247.
[6] Adams, "The Quantum Theory," *Bull. Nat. Res. Council*, 1, No. 5, Oct., 1920.
[7] Jeans, *loc. cit.*, p. 246.
[8] Ithaca, *Physic. Rev.*, **14**, p. 368.
[9] Wm. A. Felsing (Thesis), Technology Press, June, 1918.
[10] Keyes, *loc. cit.*
[11] *Proc. R. Soc. London*, **84A**, p. 595.
[12] Leipzig, *Ann. Physik*, **17A**, 1882 (418).
[13] *Proc. R. Soc. London*, **87A**, pp. 556–57.
[14] *Phil. Trans. R. Soc.*, **185A**, p. 572.
[15] *J. Chem. Soc. London*, **99**, p. 708.
[16] *Ann. Physik*, **59**, p. 200.
[17] *Landolt Bornstein* (1912).
[18] *Z. Anorg. Chem.*, **93**, p. 18.
[19] Brillouin, *Leçons sur la Viscosité*, pp. 152–159.

THE ANGULAR DIAMETER OF ALPHA BOOTIS BY THE INTERFEROMETER

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Communicated by G. E. Hale, May 2, 1921

Since the measurement of the diameter of Betelgeuse[1] in December, 1920, observations for the determination of stellar diameters have been continued with the 20-foot interferometer attached to the upper end of the 100-inch Hooker reflector.

Several definite results have been obtained, and others, less definite because of poor observing conditions, have been partially checked by comparison with stars whose diameters are known to be too small for measurement with this instrument. Let the visibility of the zero fringes for any particular separation of the mirrors be reduced by seeing which is poor as compared with that on a fine night; it is then hopeless to make final measures. If, however, the interferometer is turned to a neighboring check star and the fringes are seen, and then turned to the star under consideration and they do not appear, the observer is justified in saying that there is a definite decrease in visibility.

The data thus far obtained are as follows: For α Tauri (Aldeberan) fringes of gradually decreasing visibility have been observed at 13, 14.5 and 19 feet. Further observations are necessary between the last two positions to determine whether the fringes vanish between these points or whether a longer beam will be necessary to obtain a measure of the diameter. If the fringes vanish around 16–18 feet, as observations under poor conditions lead one to suspect they will, then the fringes observed at 19 feet must lie beyond the point of disappearance, on the ascending branch of the visibility curve where it rises toward the second maximum.

For β Geminorum (Pollux) fringes have been observed at separations of 14.5, 16, and 19 feet, the visibility being definitely reduced for these distances of the mirrors, although at 19 feet it is still considerable. A longer beam will therefore be required for the measurement of the star's diameter.

For α Cetione observation at 13 feet in variable seeing showed a reduction in visibility of the fringes when compared with that of the check star.

For α Bootis (Arcturus) observations in February and March, with poor seeing, indicated a decided reduction in the visibility of the fringes with increasing separation of the mirrors. On April 15, a series of measures was made under conditions which were excellent, as indicated by the strong contrast of the zero fringes and by diffraction rings surrounding the central disk of the star images.

At 16 feet separation of the mirrors the visibility of the fringes was estimated at 20 to 25 per cent of the zero fringes; at 17.5 feet, 15–20 per cent, certainly less conspicuous than at 16 feet. At 19.0 feet the estimate was 5 per cent, the contrast being very flat, but the fringes were picked up with certainty as the path difference was varied with the optical wedge. At 18.25 feet, the result was 15 per cent, certainly greater than at 19.0 feet; and finally at 19.5 feet no fringes whatever could be observed from the interferometer mirrors. There was no bright star in the immediate neighborhood that could be used as a check, but the instrument was known to be in adjustment for the final measure, from the certainty with which the previous settings had been made.

The separation of the mirrors for disappearance of the fringes may therefore be placed at 19.5 ± 0.5 feet, although observations with a longer beam with which the second maximum could be observed, might possibly increase this value a trifle. By disregarding any possible darkening at the limb, the result will serve to indicate the maximum diameter of the star. The spectrum of α Bootis is Ko; its approximate effective wave-length may therefore be taken at 5.6×10^{-5} cm., about halfway between that of the sun (Go) and that assumed for α Orionis (Ma).

With this value of λ and a distance of 19.5 feet (594 cm.), the angular diameter of α Bootis is 0.0237. It is interesting to note the close agreement of this figure with the estimated values of Eddington (0.020) and Russell (0.019), and with Hertzsprung's value (0.026), calculated on the basis of the measured diameter of α Orionis (0.045).

Results for the parallax of α Bootis are as follows: Yale, 0.074; Flint, 0.095; Yerkes, 0. 100; Adams, spectroscopic, 0.158; the weighted mean is 0.116. The distance of Arcturus is therefore of the order of 1.65×10^{14} miles (2.65×10^{14} km.), and its linear diameter 19×10^{6} miles (30.6×10^{6} km.), or about 22 times the diameter of the sun.

All settings of the mirrors have thus far been made by hand, and considerable time is consumed between measures. Two screws are now being mounted on the beam, driven by a single motor, which for any separation will keep the outer mirrors equidistant from the fixed inner mirrors, and thus greatly facilitate the operation of the interferometer.

[1] These PROCEEDINGS, 7, 1921 (143–146).

CHANGES OBSERVED IN THE CRAB NEBULA IN TAURUS

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Communicated by George E. Hale. Read before the Academy, April 26, 1921

Changes in the structure of the Crab nebula (N. G. C. 1952, M. 1) have recently been detected by Lampland[1] from a comparison of seventeen photographs made with the 40-inch Lowell reflector during a period of eight years. An excellent negative of this nebula had been made by Ritchey with the 60-inch Mount Wilson reflector, 1909, October 13, using a Seed 27 plate and giving an exposure of three hours with the aperture stopped down to 54 inches. For comparison with this plate, the writer made a negative with the same instrument, 1921, April 7, using full aperture and a Seed 30 plate and giving an exposure of 1 hour 20 minutes. Though the exposure of this plate was necessarily shorter than that of Ritchey's because of the proximity of the object to the sun, the larger aperture and the faster plate made the photograph comparable in density with the earlier one.

The two plates were compared in the stereocomparator, and it was seen at once that changes in the relative luminosity of different parts of the nebula had occurred, particularly in the bright region north-west of the center; and also that certain filaments and condensations had apparently moved, the motion in general being away from the center, though not always exactly along a radius.

Measures in two coördinates were made with the micrometer of the stereocomparator upon twelve nebulous condensations and thirteen comparison stars. The nebulous points chosen are sufficiently symmetrical to permit measures of about equal accuracy in the two directions. They were chosen in the outer, faint parts of the nebula because of the difficulty of seeing isolated points in the bright portions. One comparison star is the north component of the double star that lies near the center of the nebula; each of the others lies near one of the measured nebulous points. The plate constants were derived from the measures of the comparison stars by a least-squares solution; with these constants the motions of the nebulous points were then referred to the mean of the comparison stars.

The resulting displacements of the nebulous points during the $11^1/_2$ year interval are as follows:

Point	$\Delta\alpha$	$\Delta\delta$	Point	$\Delta\alpha$	$\Delta\delta$
1	+0.22	+0.70	7	−0.22	+0.11
2	+0.43	+1.24	8	+0.32	−1.56
3	−1.24	+2.05	9	+1.78	−1.78
4	−1.67	+1.24	10	+2.05	−0.27
5	−1.12	+1.59	11	+1.78	+0.70
6	−2.16	+1.19	12	+1.02	−0.05

The probable error of a measurement of a comparison star is ± 0.23 in right ascension and ± 0.25 in declination. The points measured and the observed motions are shown in Plate I, in which the lines represent the displacements that would take place in 500 years if the motion should continue at the rate indicated by the measures. The illustration shows at a glance the random distribution of the motions of the stars and the systematic character of those of the nebulous points. The mean displacements, projected upon radii drawn from the estimated center of the nebula are:

For the comparison stars,	−0.06
For the nebulous points,	+1.54

The large positive value for the nebulous points clearly indicates an outward motion, but it would be premature to suppose that this result necessarily applies to the nebula as a whole. There is some indication of a rotation in the counter-clockwise direction, but this can hardly be regarded as trustworthy.

It may be readily shown that, for a displacement of $2''$ in the interval between the two plates, the number representing the velocity in kilometers per second would be about one-fourth the number of light-years in the distance of the nebula from the solar system; for example, if the velocity were 25 km./sec. the distance would be 100 light-years. Thus it is necessary to assume neither an extraordinary distance nor an extraordinary velocity in the nebular particles in order to believe that the observed motions are real.

[1] *Harvard Observatory Bulletin, Boston*, No. 743, 1921; *Publications of the Astronomical Society of the Pacific, San Francisco*, 33, 1921 (79–84).

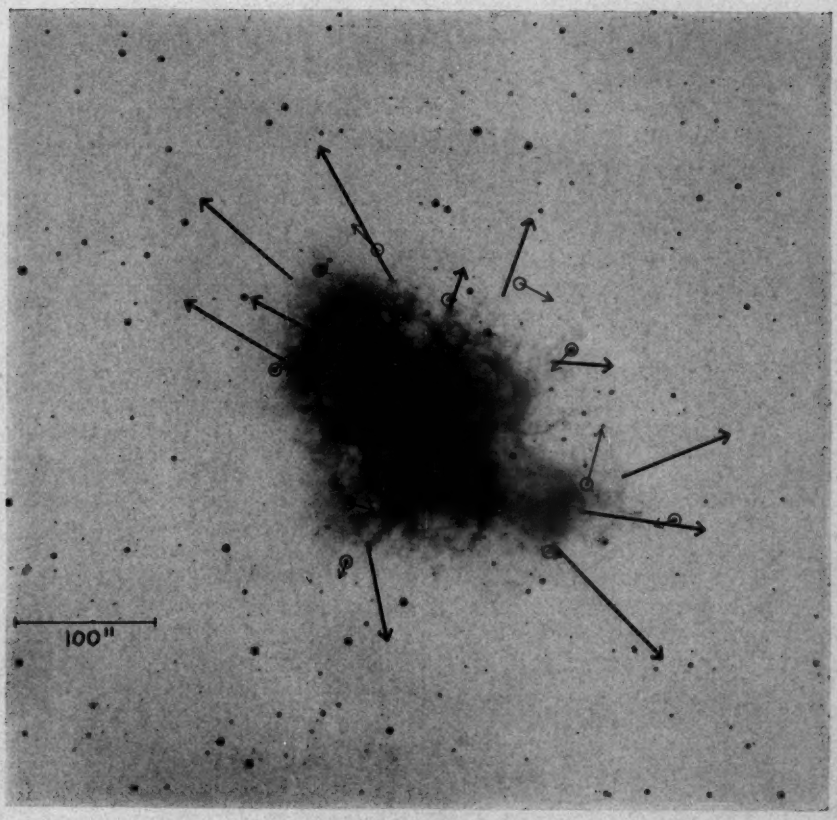

PLATE I